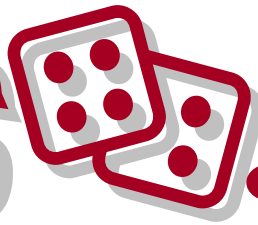
Algebraic Modelling 3 Notes...



HSC - General Maths

Name: _____

HSC CAPACITY MATRIX – GENERAL MATHEMATICS

TOPIC: Algebraic Modelling 3 & 4a – Algebra skills & Linear Modelling **2 weeks**

CONTENT	CAPACITY BREAKDOWN!	DONE IT!!!!	GOT IT!!!!!	ON MY WAY!	WORKING ON IT!	HELP!!!!
1. Using positive and negative powers of ten in scientific notation	HOTS task Ex 5A even questions					
2. Substituting into and evaluating alg expressions - linear, quadratic, cubic, square and cube roots	Ex 5B, a, c, e in each S/S task					
3. Algebraic manipulation through the 4 operations	L1: Ex 5C a, c, e, f, in each L2: Ex 5C last 4 in each question.					
4. Solving equations after substituting values	Ex 5D Q1-6, 12-14,					
5. Changing the subject of equations and formulae	Ex 5D Q7-11, 15-19					
6. Solutions of equations arising from practical situations by estimation and refinement	Ex 5E					

CONTENT	CAPACITY BREAKDOWN!	DONE IT!!!!	GOT IT!!!!!	ON MY WAY!	WORKING ON IT!	HELP!!!!
7. Generates tables of values and graph linear functions with pencil and paper	Ex 9A Q2a, d, f – generate a table for each question as well as graphing Q3					
8. Graphing point of intersection	Ex 9A 7 - 9					
9. Interpretation of the point of intersection of the graphs of two linear functions drawn from practical contexts, eg 'breakeven points' 10. Understand the term "extrapolate"	Ex 9A Q4, 6, 10, 12-14					

Your say!

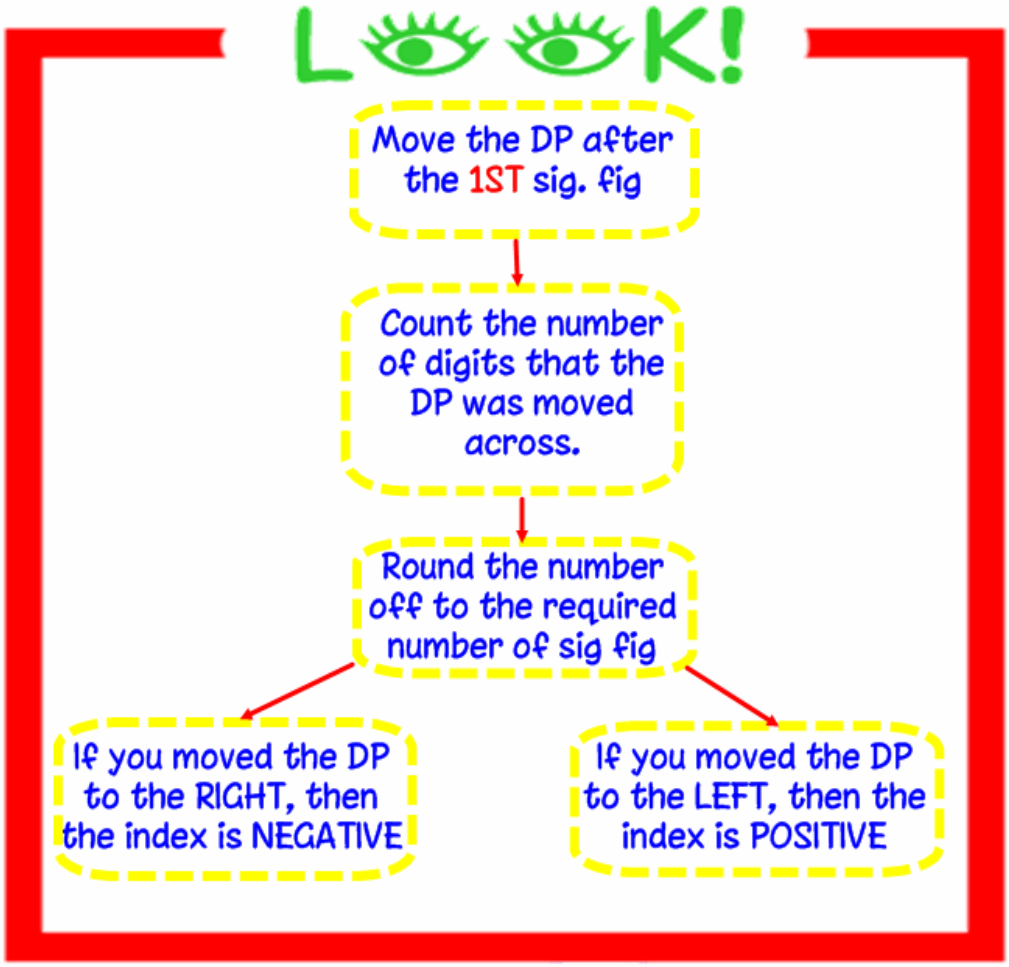
What was the most important thing you learned?_____

What was something new you learnt? _____

What part(s) of this topic will you need to work on? _____

Scientific notation and Significant figures.

Scientific notation



Do you know this number, 300 000 000 m/sec.? It's the Speed of light!

Do you recognise this number, 0.000 000 000 753 kg. ? This is the mass of a dust particle!

Scientists have developed a shorter method to express very large numbers. This method is called **scientific notation**. Scientific Notation is based on powers of the base number 10.

The number 123 000 000 000 in scientific notation is written as 1.23×10^{11}.

The first number 1.23 is called the **coefficient**.
It must be **greater than or equal to 1** and **less than 10**.

The second number is called the **base**.

It must always be 10 in scientific notation. The base number 10 is always written in exponent form (index form). In the number 1.23×10^{11} the number 11 is referred to as the exponent or power of ten.

eg Express the speed of light in scientific notation _____

Express the mass of a dust particle in scientific notation _____

Significant figures (revision....)

Every measurement has a degree of uncertainty. The number of reliably known digits in a number is called the number of **significant figures**. For example the radius of the earth is 695 000 000 m. This number is not exact but has been "rounded off" to the nearest million metres. We say the number has three **significant figures**. The zeros at the end of this number are "not significant". In some numbers zeros are significant. For example in a measurement stated as 605mm the zero is significant. The rules for determining the number of significant figures are as follows:

- All non-zero digits are significant
- Zeros between non-zero digits are significant
- Zeros at the end of a decimal are significant – **WHY?** Because the trailing zeros report a greater precision than the measuring equipment supports eg 6.500m denotes that the measure of accuracy was to the nearest mm (0.001m).
- All other zeros are not significant.

Using these rules 65.00 has four significant figures but each of the numbers 65, 6500, and 0.0065 has only two significant figures.

eg Evaluate $\sqrt[4]{\dfrac{(7.96)^8}{(5.78-2.44)^2}}$ and express your answer in scientific notation correct to two significant figures.

eg Write each of the following as a decimal number:

a) 4.576×10^{7}

b) 8.623×10^{-4}

INDICES AND SCIENTIFIC NOTATION HOTS TASK

THE HOUSE FLY

The female common house fly, *Musca domestica*, can lay up to 1000 eggs at a time. In three weeks these reach maturity and are ready to breed. Huge populations would result if all the descendants of a single pair of house flies survived and reproduced. Fortunately this is not the case as their mortality rate is very high. The few house flies we see are the true survivors.

Over the three months of summer, how many descendants could a single pair of house flies produce assuming that each pair (original and descendants) mates only once?

HAIR FACTS

There are about 1.1×10^5 hairs on your head. Each hair grows at the rate of about 1.3×10^{-3} cm per hour. A single hair lasts about six years. Every day you lose between 30 to 60 hairs. Each hair grows from a small depression in the skin called a follicle (a gland). After the hair falls out, the follicle rests for about three to four months before the next hair starts growing. Hair follicles are either oval, flat or round in shape. How straight, wavy or curly your hair is depends on the shape of your hair follicles.

How many hairs are on all the heads in China?

Draw up your family tree listing your biological parents, grandparents and great grandparents.
Can you see a pattern occurring in the number of parents, grandparents and so on?

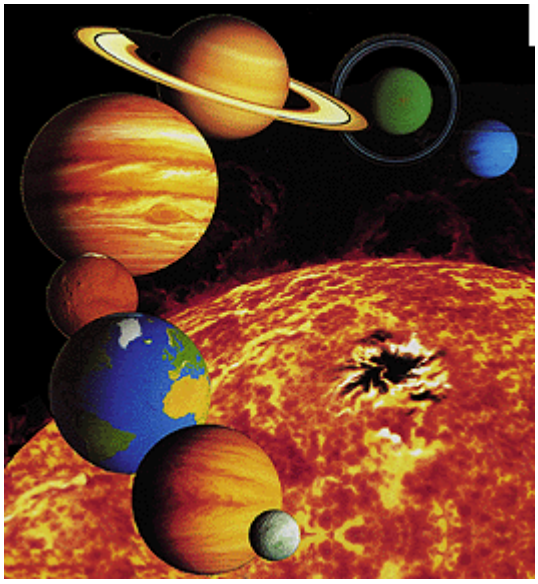
Calculate the number of

a) Great great grandparents you had
b) People in your tree going back 6 generations

and justify your answers.

solar system scale model

Planet	Average Distance from sun	Average Length of orbit (km)	Length of year	Average Speed (km/day)
Mercury	5.8×10^7		88 days	
Venus	1.08×10^8		224.7 days	
Earth	1.5×10^8		365.26 days	
Mars	2.28×10^8		1.88 yrs	
Jupiter	7.78×10^8		11.86 yrs	
Saturn	1.427×10^9		29.46 yrs	
Uranus	2.87×10^9		84.01 yrs	
Neptune	4.497×10^9		164.79 yrs	
Pluto	5.9×10^9		247.7 yrs	

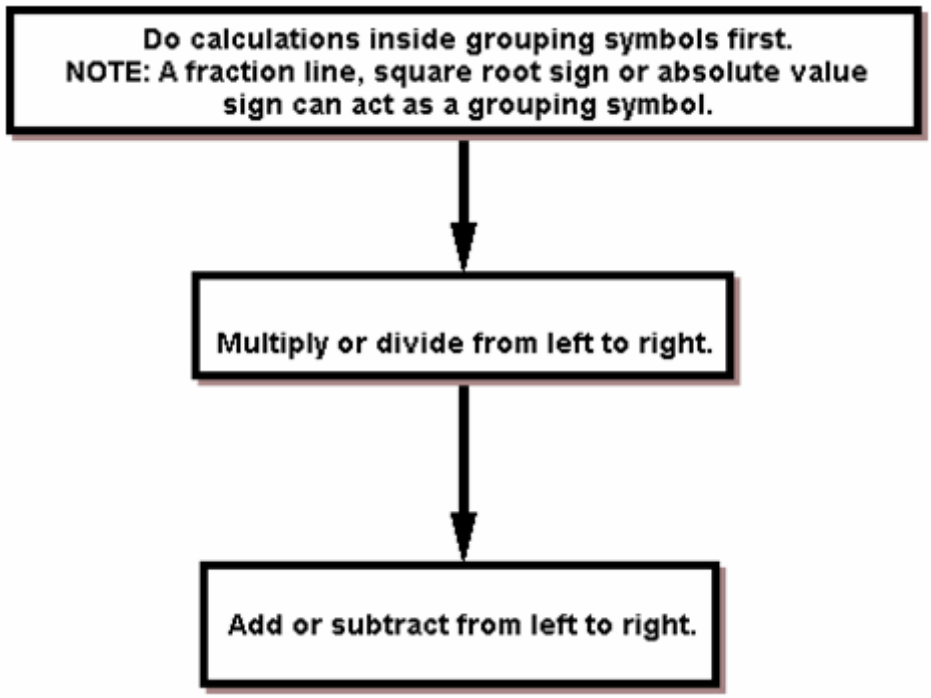


Substitution...

Substitution involves replacing a pronumeral in an expression with a numerical value then solving for the unknown pronumeral.

Setting out is important:

- **Write the expression;**
- **Substitute all known values;**
- **Follow order of operations to calculate the value of expression.**



eg The expression $r = \sqrt[3]{\frac{3V}{4\pi}}$ calculates the radius of a sphere, given the volume. Find the radius of a sphere with a volume of 350 cm^3. Give your answer correct to 1 decimal place.

Algebra...A review!

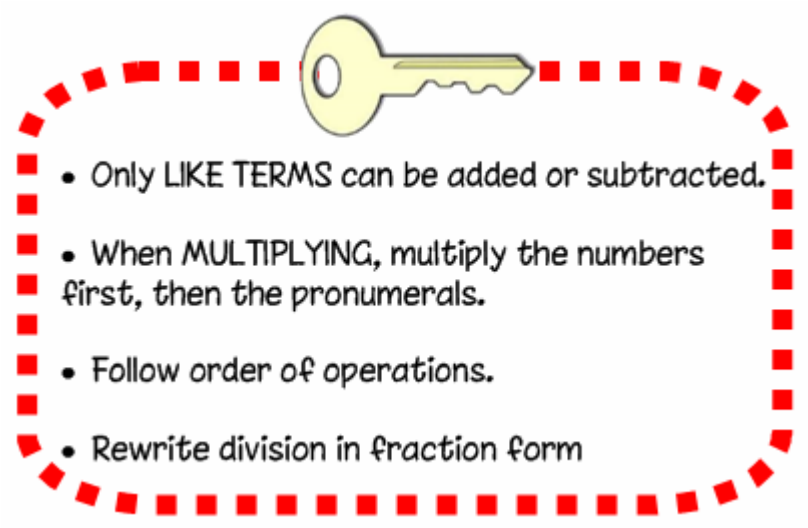
Simplifying...

What's a like term? _____

Write 3 different like terms for the term $2x^2y$: _____

Circle the terms in the following expression:

$$5ws + 4w - 6wy^2 + y$$



Simplify the following:

a) $5x^2 + 5x - x^2$

b) $5ab - 4ba$

c) $4x^3 + x + 6x^2 - 3x - x^2$

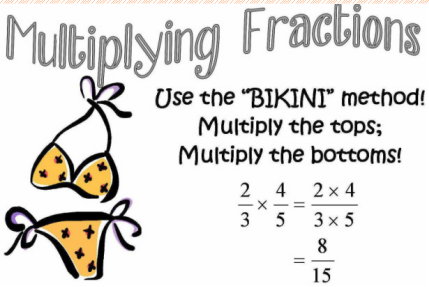
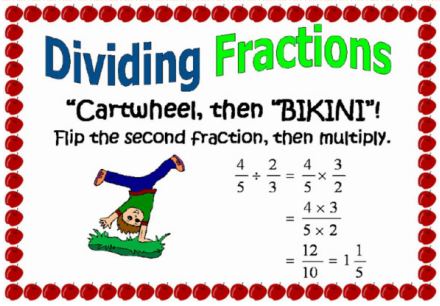
d) $-6ab \times -5b$

e) $(-3xy)^2$

f) $-12xy \div 4$

Algebriac Fractions...

APPLY THE FRACTION RULES YOU LEARNT IN YEAR 7!

Addition & Subtraction	Multiplication	Division
	 Also remember to check diagonals to cancel any common factors.	

Examples

a) $\frac{5x}{6}+\frac{3x}{4}=$

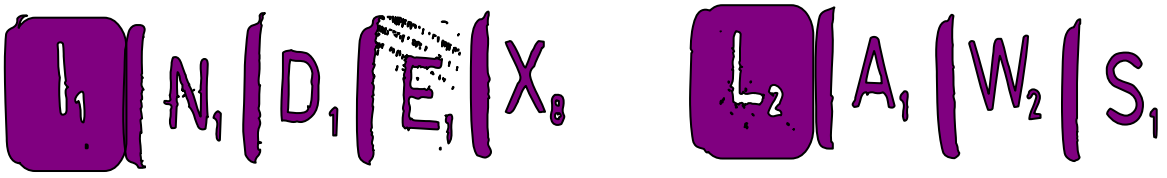
b) $\frac{x+2}{3}-\frac{x}{4}=$

c) $\frac{3m}{5}\times\frac{7a}{4}=$

d) $\frac{11ab}{7xy}+\frac{14x}{5b}=$

e) $\frac{2x}{5}\div\frac{3}{4}=$

f) $\frac{5m^2}{7}\div\frac{m}{2}=$



$$x^m \times x^n = x^{m+n}$$

$$x^m \div x^n = \frac{x^m}{x^n} = x^{m-n}$$

$$\left(x^m\right)^n = x^{mn}$$

$$x^0 = 1$$

$$x^{-m} = \frac{1}{x^m}$$

$$x^{\frac{m}{n}} = \sqrt[n]{x^m}$$

$$\left(\sqrt{x}\right)^2 = x$$

Examples:

a) $9a^4 \times 8ab^3 =$

b) $(3x^3)^4 =$

c) $32m^6n^2 \div 4m^2n^3 =$

d) $\left(\frac{2a}{3}\right)^{-2} =$

e) $16x^4 \div 12x^6 =$

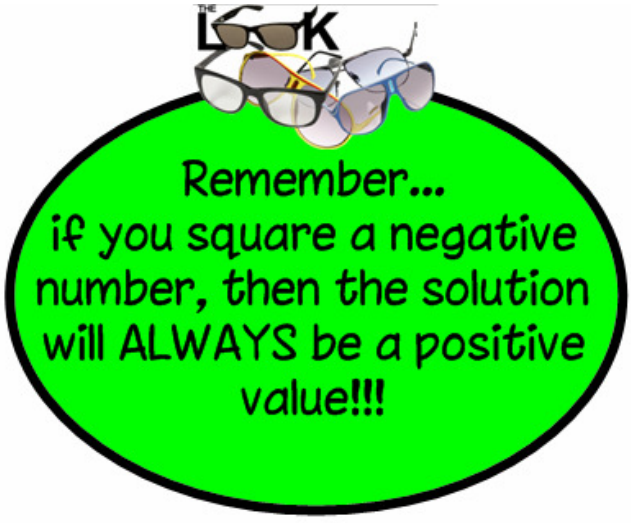
f) Remove the fraction format

$$\frac{8x^2}{y^3} =$$

g) Rewrite in index form $7(x-4)^{-1} =$

h) $(-2w^2)^2 =$

i) $-(2w^2)^2 =$

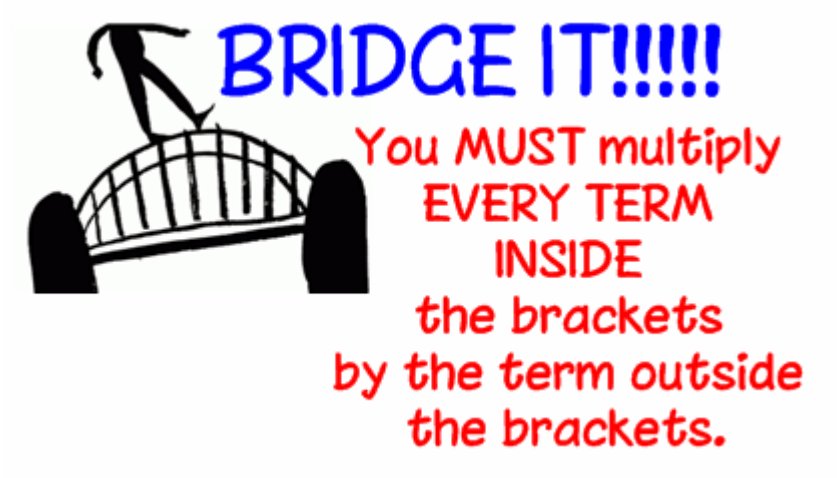


j) $a^3bc^2 \div 2ac^3 =$

k) $8m^2n \div 32mn =$

l) $(-x)^3 =$

Removing grouping symbols...



Expand & simplify the following:

a) $2(x+3)=$

b) $5a(a-7)=$

c) $2p(3p+1)-5p=$

d) $2(6a-7)+(8a-3)=$

e) $(9m-8)-(2m-5)=$

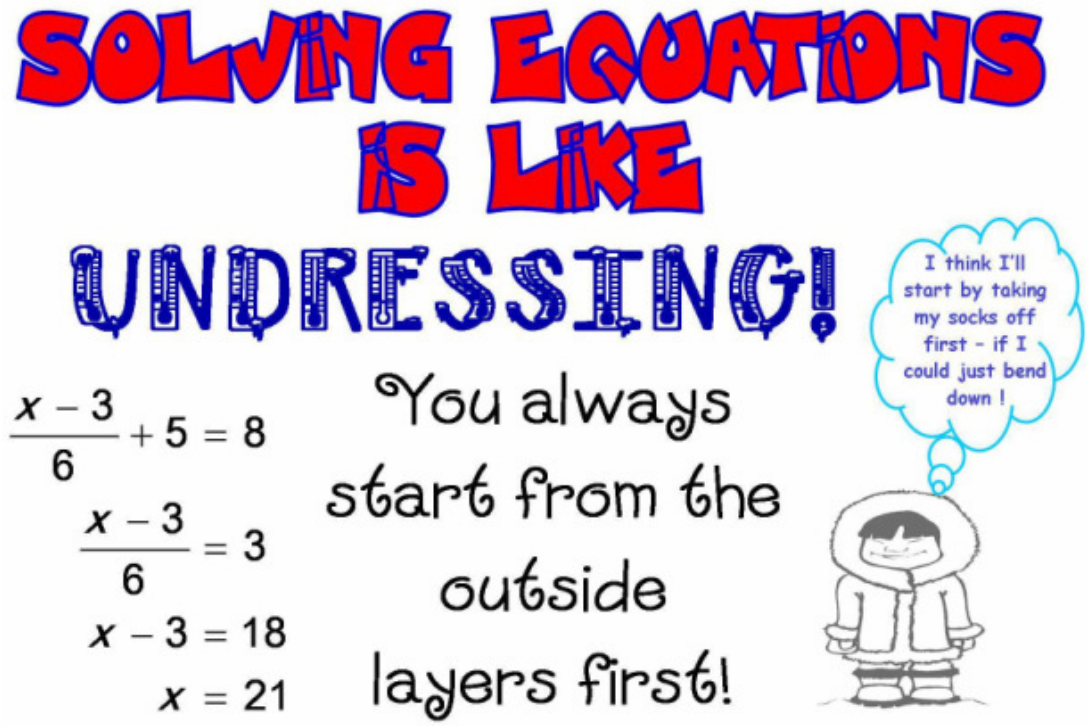
f) $9-(a-b)-(a+b)=$

g) $7(a-3)-2(3a-1)=$

Solving equations...

An equation is different to an expression in that it has both a left and right hand side. It is formed after substituting into a formula.

To solve an equation, use the opposite operation and keep the following in mind!

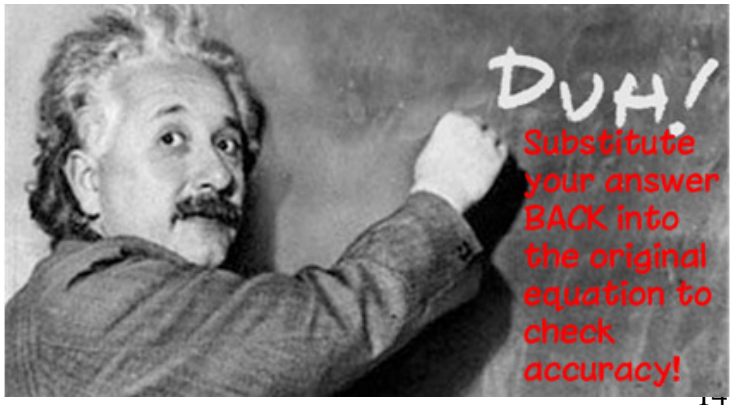


eg The product of m and k is 5 less than y can be written as:

(A) $m + k = 5 - y$ (B) $m + k = y - 5$ (C) $mk = 5 - y$ (D) $mk = y - 5$

eg Given $E = \frac{1}{2}mv^2$, calculate the value of m when E = 730.08 and v = 7.8

eg Solve $\frac{2x-5}{3} = \frac{x}{6} + 2$



Changing the Subject...

http://www.bbc.co.uk/schools/gcsebitesize/maths/algebra/formulaehirev1.shtml

It's like peeling an onion - remove each layer, one by one.

THE ORDER IN WHICH YOU DO THIS IS IMPORTANT!

At each stage, STOP AND THINK, What is the operation that is on the outside

SOME TRICKS TO USE:

Remove denominators ASAP!
Expand brackets IF IT HELPS!!!!

eg Change the subject for the following to the letter in the brackets:

$w = \frac{y-3}{2}$ [y]

$A = \pi r^2$ [r]

$L = \frac{1}{2}(h - t)$ [h]

$P = 4 + \frac{5}{w}$ [w]

$v = u + at$ [t]

For equations that have the variable in the index position, use your LOG button!

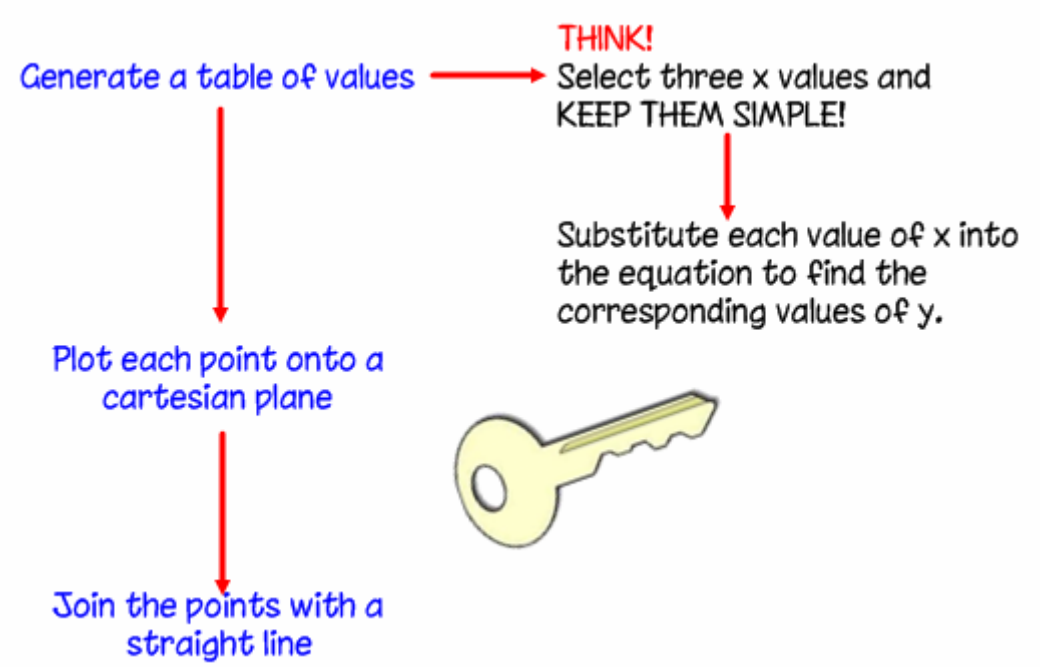
Finally a use for that button!!!

For $a^x = b$, $x = \dfrac{\log(b)}{\log(a)}$

eg Find an approximate solution to the equation $3^x = 100$

Graphing Linear Functions...

What's a linear function?



Gradient...

Revision…

What are the different ways you can find the gradient of a linear function?

Points of Intersection...

When two linear functions are graphed on the same pair of axes, the point of intersection informs you of what?

eg The cost of producing shoes in Asia is given by the equation C = 2000 + 15n where n is the number of pairs of shoes produced each day. The cost of producing shoes in Australia is given by the equation C = 1000 + 20n.

a) On the same pair of axes, graph the cost equations for producing shoes in Asia and Australia.

b) When is it more cost efficient to produce the shoes in Asia?

1. Draw up tables of values for each cost equation

2. Plot the points onto the same plane (label each line with its function)
3. Consider your graphs and answer the final question.